

MATHEMATICAL MODELING IN ENGINEERING CONTROL

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Abstract— The role of mathematics in control systems is very crucial in understanding various real time systems. This paper elaborates on some of the classical techniques used in control design applied to certain non linear systems.

Keywords— Dynamics, control, differential equations.

I. INTRODUCTION

A dynamical system is a system whose behavior changes over time, often in response to external stimulation or forcing. The term feedback refers to a situation in which two (or more) dynamical systems are connected together such that each system influences the other and their dynamics are thus strongly coupled [1]. Feedback however has a potential disadvantage. It can create dynamic instabilities in a system, causing oscillations or even runaway behavior. Another drawback, especially in engineering systems, is that feedback can introduce unwanted sensor noise into the system, requiring careful filtering of signals. It is for these reasons that a substantial portion of the study of feedback systems is devoted to developing an understanding of dynamics and a mastery of techniques in dynamical systems. The role of mathematics in developing an understanding of dynamics and a mastery of techniques in dynamic systems is very crucial.

In modern control systems, mathematical computation is typically implemented on a digital computer, requiring the use of analog-to-digital (A/D) and digital-to-analog (D/A) converters. Uncertainty enters the system through noise in sensing and actuation subsystems, external disturbances that affect the underlying system operation and uncertain dynamics in the system (parameter errors, unmodeled effects, etc). The algorithm that computes the control action as a function of the sensor values is often called a control law. The system can be influenced externally by an operator who introduces command signals to the system.

II. CURRENT TREND

Many problems in the natural sciences involve understanding aggregate behavior in complex large-scale systems. This behavior emerges from the interaction of a multitude of simpler systems with intricate patterns of information flow. Two specific methods commonly used in feedback and control systems are differential equations and difference equations.

III. MODEL DEVELOPMENT

A model is a mathematical representation of a physical, biological or information system. A common class of mathematical models for dynamical systems is ordinary differential equations (ODEs).

A generic second order forced or controlled differential system can be represented by

$$\ddot{y} + a\dot{y} + by = r(t) \quad (1)$$

where $y(t)$ is the output and $r(t)$ is the input. Ordinary differential equations are not suitable for component based modeling as internal description of a component may change when it is connected to other components. This difficulty can be avoided by replacing differential equations by differential algebraic equations, which have the form

$$F(Z, \dot{Z}) = 0 \quad (2)$$

where $z \in R_n$ and R_n is the set of real numbers.

A simple way to determine a system's dynamics is to observe the response to a step change in the control signal. Such an experiment begins by setting the control signal to a constant value. Then when steady state is established, the control signal is changed quickly to a new level and the output is observed. The experiment gives the step response of the system, and the shape of the response gives useful information about the dynamics. A simple voltage divider circuit for various types of load with a sinusoidal source is discussed in this paper.

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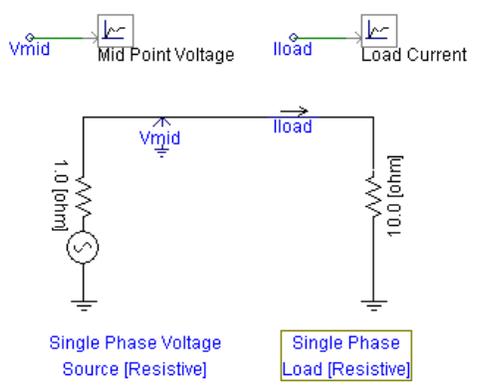


Fig. 1 Voltage divider circuit with resistive load

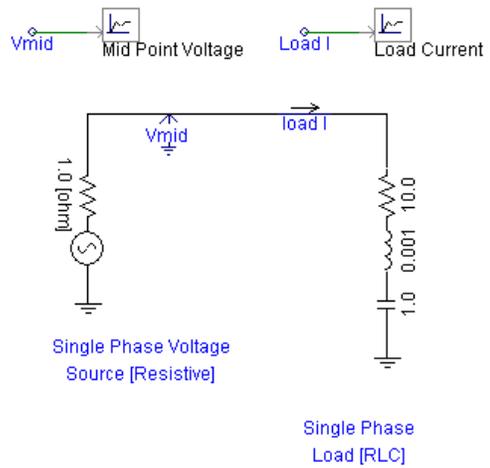


Fig. 4. Voltage divider circuit with RLC Load

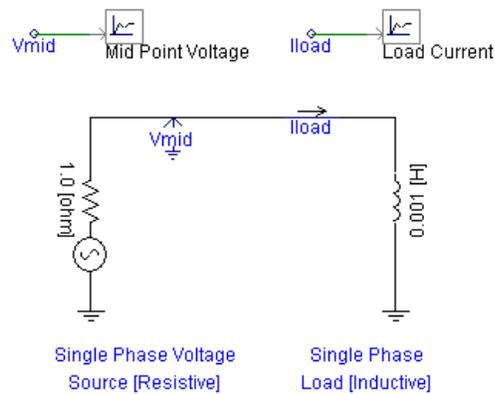


Fig. 2 Voltage divider circuit with inductive load

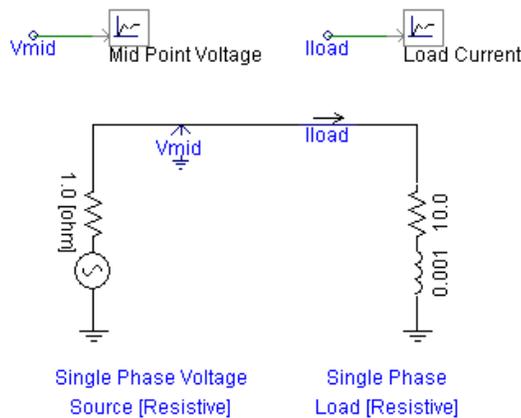


Fig. 3 Voltage divider circuit with RL load

IV. RESULTS AND DISCUSSION

The PSCAD simulation results for voltage divider circuit with various types of loads are as given below. In Fig5. Midpoint voltage and current for a resistive load of 10 Ohm is shown. In Fig6. Midpoint voltage and current for an inductive load of 0.001H is shown. Fig7. Shows midpoint voltage and current in case of a RL load with Resistance of 10 Ohms and inductance of 0.001H. A zero valued current is shown for a RLC circuit of 10 Ohms, 0.001H & 1F.

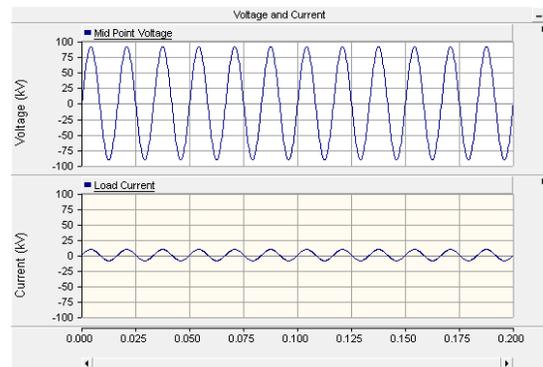


Fig 5. Mid Point voltage & current -R load.

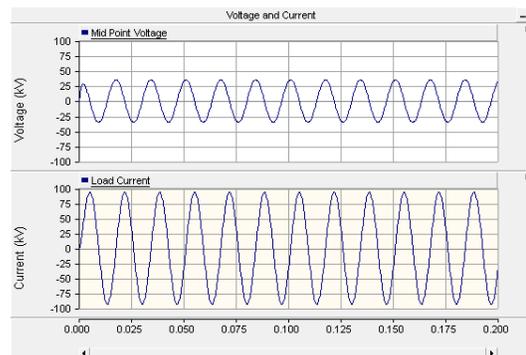


Fig 6. Mid Pt voltage & current -inductive load.

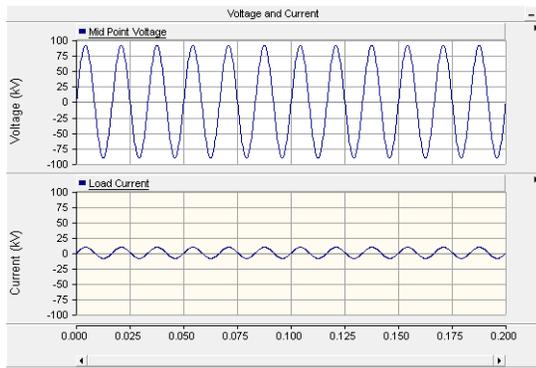


Fig7. Mid Pt. voltage & current -RL load.

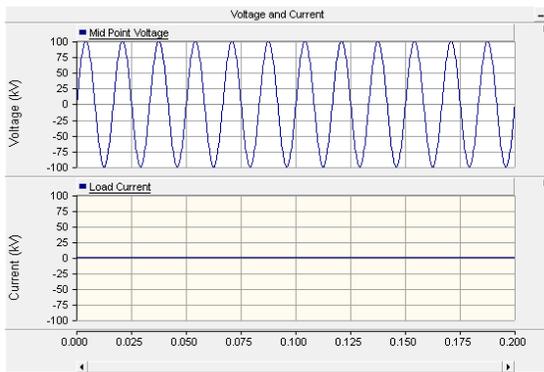


Fig8. Midpoint voltage & current-RLC load.

V.CONCLUSIONS

Based on the simulations performed, it was observed that when the load was changed from a resistive load of 10 ohms to an inductive load of 0.001H, the control sequence has changed from voltage to current. And further when a capacitor was added to the circuit, the current died down to a value of zero. The underlying mathematical phenomenon of an equation changing from a simple linear equation to a first order equation and then to a second order equation shows that the system response dies down as the order of the system increases.

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